CALCULATING ψ and Δ USING A PHOTOELASTIC MODULATOR FROM A LOCK-IN AMPLIFIER

It is possible to measure the angles ψ and Δ using a photoelastic modulator and lock-in amplifiers. The block diagram below is a possible configuration for using PEMs and lock-ins to make ellipsometric measurements.

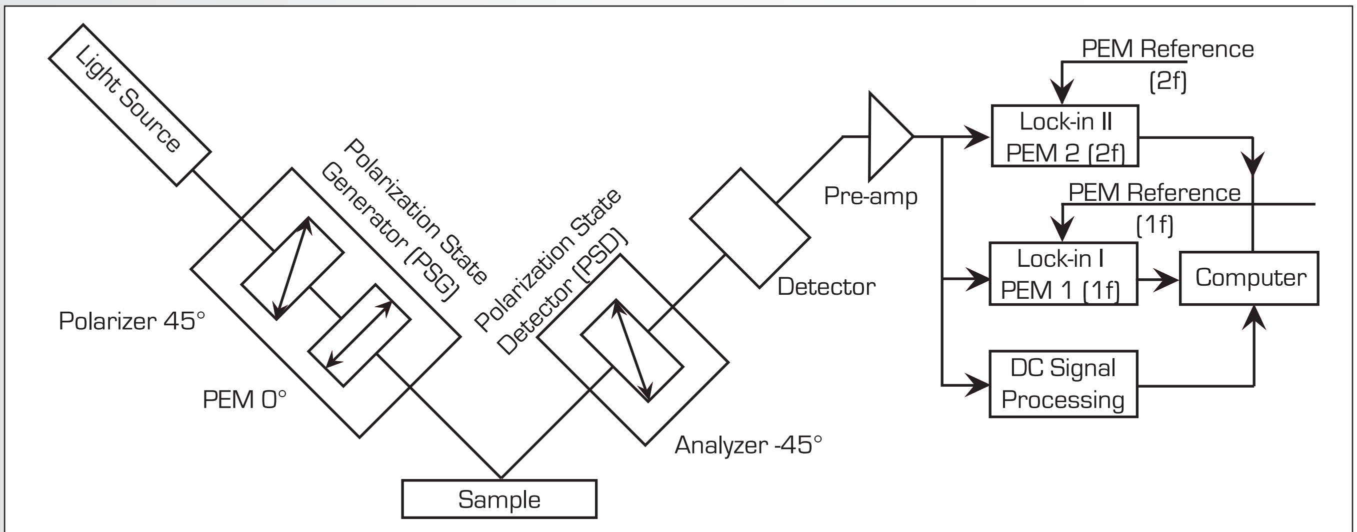

FIGURE 1. A PEM ELLIPSOMETER WITH LOCK-IN DEMODULATION

Lock-in amplifiers give us voltage readings. These readings can be taken and transferred to obtain the ellipsometric angles of ψ and Δ. To begin, a measurement of the intensity (voltage) of the 1f, 2f, and DC should be measured. The Signal conditioning unit can be used to isolate the DC and the AC signals.

The ratio of the AC to DC signals can then be seen as:.

$$\frac{I_{1f}}{I_{DC}} = 2J_1(A)\cdot(I_X - \delta_0 \cdot I_Y)$$

and

$$\frac{I_{2f}}{I_{DC}} = 2\cdot J_2(A)\cdot(I_Y - \delta_0 \cdot I_Y)$$

It is important to note that most lock-in amplifiers measure the RMS value of a signal. However, what is needed for this calculation is the peak-to-peak voltage value. Therefore the AC to DC ratios will need to be multiplied by $\sqrt{2}$.

The term δ_0 is the static retardation of the PEM parallel to (or perpendicular to) the PEM retardation axis. This is assumed to be small, and it is for Hinds PEMs.

These have been solved for I_X and I_Y below. Terms in δ_0^2 have been neglected, since δ_0 was assumed small to begin with.

$$I_X = \frac{1}{2J_1(A)} \cdot \frac{I_{1f}}{I_{DC}} + \frac{\delta_0}{2J_2(A)} \cdot \frac{I_{2f}}{I_{DC}}$$

and

$$I_Y = \frac{1}{2J_2(A)} \cdot \frac{I_{2f}}{I_{DC}} + \frac{\delta_0}{2J_1(A)} \cdot \frac{I_{1f}}{I_{DC}}$$

Again, if a lock-in amplifier is used that displays RMS values, the ratios in the above equations should be multiplied by $\sqrt{2}$.

The quantity A is the Bessel angle of the PEM modulation and is proportional to the angle of modulation. Typically, this is set to 2.4048 radians. This is done so that $J_0(A) = 0$. With this simplification in mind, $J_1(A) = 0.519$ and $J_2(A) = 0.432$.

Since all other terms in the equation are known, the values of I_X and I_Y can now be calculated. Once these values are known, it is possible to calculate the terms, N,S, and C, which are trigonometric relations to ψ and Δ.

I_X, I_Y and I_{DC} can be expressed in the following equations.

$$I_{DC} = 1 - N \bullet \cos(2\theta_a)$$

$$I_X = S \bullet \sin(2\theta_a)$$

$$I_Y = \sin(2\theta_m) \bullet (\cos(2\theta_m) - N) - C \bullet \cos(2\theta_m)\sin(2\theta_a)$$

θ_a is the athmuzal angle of the polarization state analyzer and θ_m is the athmuzal angle of the polarization state generator. The above equations can be greatly simplified by setting θ_a to -45° (i.e. setting the analyzer that is placed before the detector to - 45°). These equations then reduce to:

$$I_{DC} = 1$$

$$I_X = S$$

$$I_Y = -N \bullet \sin(2\theta_m) - C \bullet (\cos(2\theta_m)$$

The quantity S is then known because I_X was calculated above.

N and C are calculated based upon the angle, θ_m. If θ_m is set to 45° (i.e. the polarizer preceding the PEM is set to 45°, and the PEM is set to 0°), the cosine term, and thus the C variable goes to 0. Therefore, N can be calculated. If the polarizer is set to 0°, then the sine term, and thus the N term, goes to 0 and the C term can be calculated.

The three terms N, S, and C are related by a linear equation, $N^2 + S^2 + C^2 = 1$. However, this equation is entirely dependant upon there being no depolarization effects in the sample being measured. This equation does not hold true if the sample is a depolarizing sample.

This equation can be, however, used to improve the accuracy of the N, S, and C measurements. If all three values are measured and then 2 values are used to calculate the third value, a ratio of the measured value to the calculated value can be taken. If this is performed for all three variables, it has the effect of normalizing the N, S, and C values and produces more accurate results.

The values N, S, and C are related to ψ and Δ by:

$$N = \cos(2\psi) \quad S = \sin(2\psi)\sin(\Delta) \quad C = \sin(2\psi)\cos(\Delta) .$$

Either the measured, calculated, or a ratio of the 2 values of N, S, and C can be plugged into the above equations to generate ψ and Δ. If N and S are known, ψ can easily be calculated by plugging in the value for N. Δ can then be calculated by plugging in values for ψ and S. If S and C are know, ψ and Δ can be solved by plugging in values for S and C and using trigonometric functions to solve for the angles.

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