BASICS: FOURIER TRANSFORM SPECTROMETRY

The ARCspectro-NIR (MIR) and ARCspectro-HT manufactured by ARCoptix are Fourier transform spectrometers (FTS). Although this type of device is used for the same scope as more conventional grating spectrometers - i.e. analyzing the spectrum of light - FTS operates in a different manner. This document describes their basic working principle in a concise and didactical manner.

HOW DOES IT WORK?

A FT spectrometer is an interferometer. Its historical and most widely known configuration is the *Michelson interferometer,* shown on Fig. 1. A beam-splitter is used for dividing the light to be spectrally analyzed into two beams. After they have been reflected on two distinct mirrors, the two beams are recombined by the same beam-splitter and sent to a detector. One of the mirrors is fixed, while the other is a movable mirror. When the fixed and the movable mirrors are equidistant from the beam splitter (d=0) both beams travel the same distance before reaching the detector. However, if the movable mirror moved away, one of the light beams has to travel an additional distance **δ = 2d** (back and forth) called ***retardation***, or optical path difference.

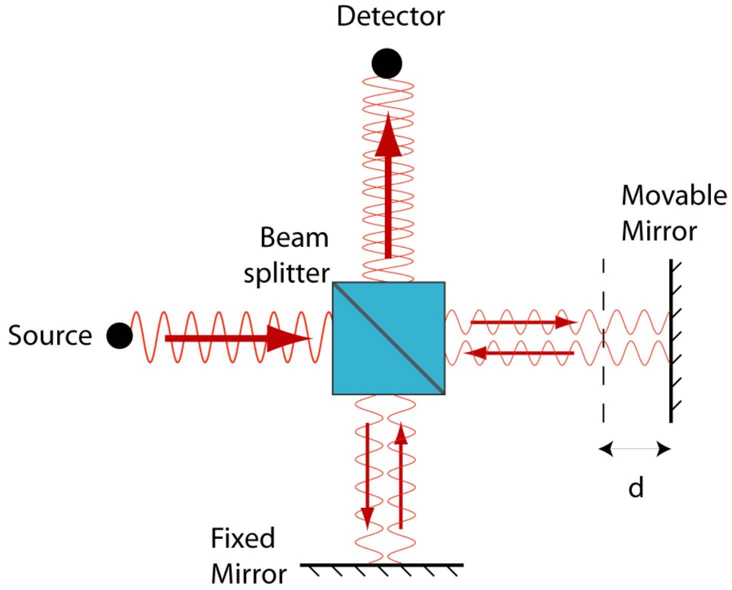

Fig. 1 - The Michelson interferometer

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MONOCHROMATIC LIGHT - INTERFERENCE

Let us first consider a perfectly monochromatic light source. At the detector, the recombined beams will produce interference, which can be constructive or destructive. If $\delta = 0$ (i.e. if the two mirrors are equidistant from the beamsplitter) or if δ is an integer multiple of the wavelength λ ($\delta = n\lambda$, with n integer) then the two beams are said to be ***in phase*** are and ***constructive*** interference is produced: the intensity at the detector is equal to the intensity of the source.

On the opposite, if $\delta = \lambda/2$ or if δ is equal to $\delta = \lambda/2$ plus an integer multiple of the wavelength λ (i.e. if $\delta = (n+1)\lambda/2$) then the two beams are said to be in ***out of phase*** are and ***destructive*** interference is produced: the intensity at the detector is zero.

For intermediate retardations δ, a smooth behaviour takes place. By scanning the movable mirror over some distance, the intensity $I(\delta)$ recorded on the detector is a sinusoidal function:

$$I(\delta) = \frac{I_0}{2}\left[1 + \cos\left(\frac{2\pi}{\lambda}\delta\right)\right]$$

Where I_0 is the intensity of the source. Graphically, it looks like this:

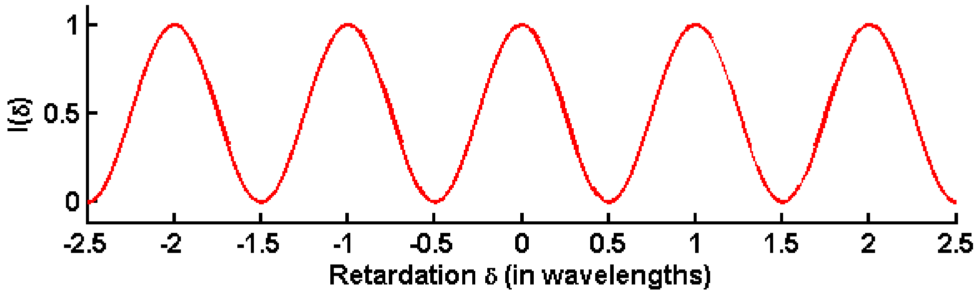


Fig 2 – Monochromatic light interferogram

The intensity $I(\delta)$ measured as a function of the retardation δ is called the ***interferogram***. It is important to understand that the period in the recorded interferogram depends on the light wavelength: the longer is the wavelength, the larger is the period in the interferogram.

BI-CHROMATIC LIGHT

Let us consider a slightly more complicated situation where the light source emits at two discrete wavelengths, which is illustrated on Fig. 3. The left graph schematically illustrates the spectra of such a bi-chromatic light. We have assumed that the shorter wavelength (blue) is less intense than the longer wavelength (red).

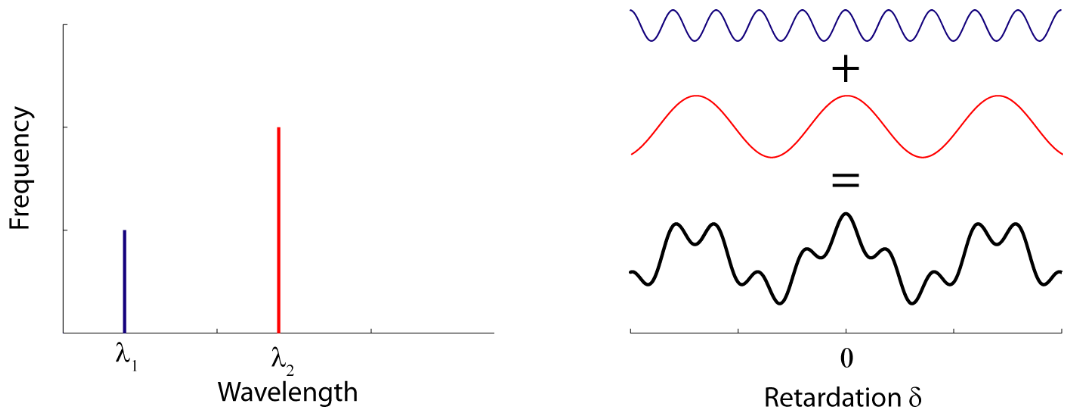


Fig. 3 – Schematic representation of an interferogram produced by bi-chromatic light

The right-hand graph in Fig. 3 shows three interferograms. The upper interferogram is the one produced by the spectral line with shorter wavelength and weaker intensity (upper blue). It has a short period and small amplitude. The interferogram produced by the spectral line with longer wavelength and higher intensity (middle red) has a longer period and larger amplitude. Due to the linearity of the process, the interferogram that is effectively recorded by the photo-detector is simply the sum of the two other interferograms (lower black curve).

In the example above, the two original wavelengths can still be distinguished quite clearly in the composed interferogram. However, in more complex circumstances, it is usually not so evident. A mathematical operation is needed for identifying the spectral intensity of the source starting from the measured interferogram - called Fourier transform - which will be discussed further.

POLYCHROMATIC LIGHT

When the light source is characterized by a ***wide-band spectrum*** things are not so much different. For example, consider the spectrum shown in Fig.4. Each spectral component of the light source can be thought of producing an interferogram with its characteristic period, and whose amplitude is weighted by the relative spectral intensity.

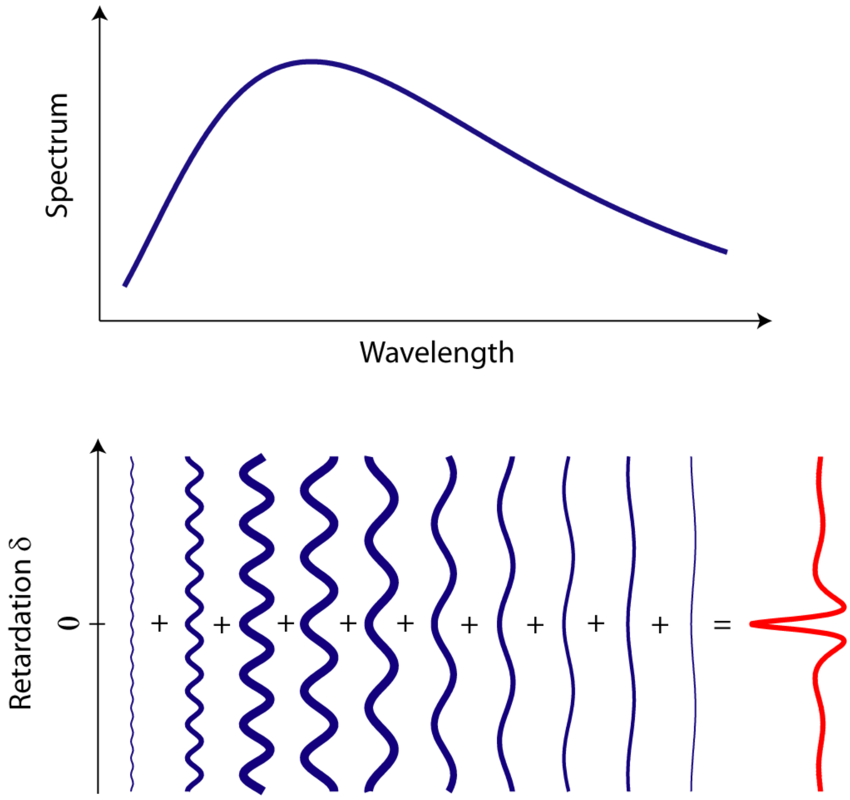


Fig. 4 – Schematic illustration of the formation of a broad-spectrum interferogram

Again, the interferogram recorded by the detector is simply the sum of all these weighted monochromatic interferograms (red line). Note that all wavelengths are simultaneously interfering constructively at the zero retardation point ($\delta = 0$). This causes a strong intensity peak in the compound interferogram, commonly called *centerburst*, which is typical of interferograms recorded with wide-band spectra. At larger retardations, the modulation in the interferogram progressively dies-out. Indeed, the maxima of the monochromatic interferograms are progressively going out of phase, thus they are cancelling each other.

FOURIER TRANSFORM

Mathematically speaking, the sum of the monochromatic interferograms simply turns into an integral (i.e. not a discrete but a continuous summation). The recorded interferogram can by calculated by:

$$I(\delta) = \int_{-\infty}^{+\infty} dv\, S(v) \cos(2\pi v \delta)$$

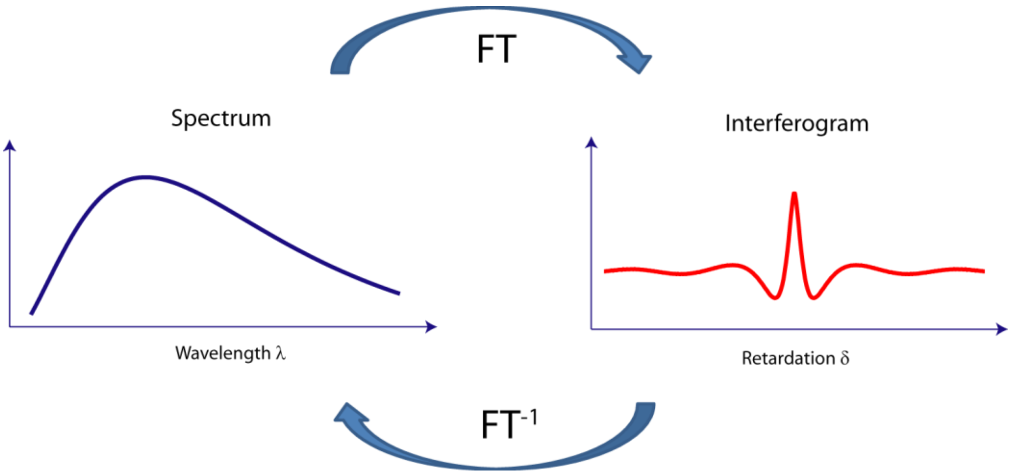
$where\ S(v)$ is the spectrum of the light source expressed in wave-numbers $v = 1/\lambda$ (the inverse of the wavelength, the so-called *wave-number* or *spatial frequency*). Do not bother too much on the fact that the spectrum is expressed in terms of wave-numbers instead of wavelengths. The only reason is that the integral, called ***(cosine) Fourier transform***, has a slightly simpler mathematical form when expressed in wave-numbers.

Getting the spectra from the interferogram

Usually, one is interested in the spectrum of the light source producing the interferogram. The Fourier transform has the useful property of having an *inverse*, that allows calculating back the spectrum $S(v)$ from the interferogram $I(\delta)$. The inverse cosine Fourier transform is very simply expressed by:

$$S(v) = \int_{-\infty}^{+\infty} d\delta\, I(\delta) \cos(2\pi v \delta)$$

In conclusion, the essential idea to remember is that there is a 1-to-1 correspondence between spectra and interferogram. Each particular light spectrum is related to a unique interferogram, and each interferogram corresponds to a unique spectrum. The mathematical relationship between the two is the Fourier transform.



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